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National Aeronautics and Space Administration
Goddard Space Flight Center
Contract No. NAS-5-9299

ST - RWP - CR - 10449

DEPOLARIZATION OF COSMIC RADIO EMISSION DUE TO DISPERSION OF FARADAY ROTATION OF RADIOWAVE POLARIZATION PLANES

by

V. A. Razin
V. V. Khrulev

(USSR)

22 FEBRUARY 1966

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IV UZ, Radiofizika
Tom 8, No. 6, 1063-1068,
Izd. Gor'kovskogo Univ. 1965

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SUMMARY

This paper studies the depolarization of cosmic radio emission caused by the dispersion of Faraday rotation of radiowave polarization planes and the finite width of the apparatus' pass band.

General expressions have been derived, which link the position angle and the degree of polarization of radiowaves with the frequency characteristics of the receiver.

The results of calculations of the degree of polarization are given in the form of formulas and graphs for a series of prevalent circuits. So long as the degree of polarization has a notable value ($\geqslant 0.05$), the position angle varies only by a few degrees when the pass band width of the receiver varies by $1 \div 2\%$ of resonance frequency.

* * *

When the linearly polarized radio emission propagates in the interstellar medium, in interplanetary space and terrestrial ionosphere, the polarization planes of radiowaves undergo the rotation phenomenon (Faraday effect). The rotation angle of the polarization plane is inversely proportional to the square of the frequency; this is why at sufficiently great value of the Faraday rotation the radiowaves with frequencies within the range of apparatus' pass band are found to be polarized in different planes at the reception spot, and the radio emission is depolarized [1 – 3]. The knowledge to what extent the polarization of cosmic radio emission is then decreased is prerequisite for a correct selection of the width of receiving apparatus' pass band, particularly when the polarized measurements are conducted by the method of latter's modulation [2 – 3]. Besides, knowing the

* Depolyarizatisya kosmicheskogo radioizlucheniya iz-za dispersii faradyevskogo vrashcheniya ploskostey polyarizatsii radiovoln.

dependence of the degree of polarization on the pass band width of the apparatus, it is possible to determine the magnitude of the Faraday rotation of the polarization plane of cosmic radio emission over its path from the sources to the observer [4].

The depolarization of radio emission on account of dispersion of the Faraday effect has been discussed in the works [2 – 5]. Several simplest cases were considered in these works : the frequency charactristic of the apparatus was approximated by the resonance curve of a unique circuit [2, 3], by the Gaussian curve [4], assumed rectangular [5], and the radiation intensity in the apparatus' pass band was at the same time estimated constant. This question is analyzed in this work at further length. The degree of polarization of radio emission is, in particular, computed in the assumption that the apparatus' pass band is determined by a few cascades tuned to a single frequency, or by groups of two or three mutually detuned cascades.

It is rather difficult to compute in its general form the factor determining the depolarization of cosmic radio emission on account of the Faraday effect dispersion, for the polarization planes of radiowaves incident from various directions within the bounds of the main lobe of antenna radiation pattern rotate by various angles. It should however be taken into account that the difference $\Delta\Psi$ of rotation angles of waves' polarization planes, with frequencies differing by $\Delta\nu$ is equal to $2\Psi_0\Delta\nu/\nu_0$ in the case when $\Delta\nu \ll \nu_0$, ν_0 being the mean frequency), and is much less than Ψ_0, which is the rotation angle of the polarization plane of radio emission with frequency ν_0. Differences of the order of one radian in Ψ_0 affect little the quantity $\Delta\Psi$. That is why we shall postulate the following: 1) in the Galaxy region the Faraday rotation is absent; 2) the antenna radiation pattern is so narrow that the difference in the rotation angles of polarization planes of waves incident from various directions may be neglected. We shall moreover assume that the effective spectrum width of the received radio emission is

$$(\Delta\nu)_{\text{эфф}} \ll \nu_0.$$

The Stokes parameters for a linearly polarized cosmic radio emission in the interval of frequencies $d\nu$ will be written in the form

$$
\begin{aligned}
dI &= F(\nu)\; T(\nu)\; d\nu, \\
dQ &= F(\nu)\; T(\nu) \cos(2\chi(\nu))\; d\nu, \\
dU &= F(\nu)\; T(\nu) \sin(2\chi(\nu))\; d\nu, \\
dV &= 0,
\end{aligned}
\tag{1}
$$

where $F(\nu)$ is the energetic frequency characteristic of the receiving apparatus, $T(\nu)$ is the equivalent brightness temperature of the linearly polarized component of cosmic radio emission in the frequency ν, $\chi(\nu)$ is the radiation's position angle in the frequency ν. The Stokes parameters of radio emission received by the apparatus are found by way of integration of the expressions (1) over the frequency.

Taking into account the above assumptions, and assuming also that $T(\nu) \sim \nu^{-\alpha}$, where α is the spectral index, we may write the following expressions for $\chi(\nu)$ and $T(\nu)$:

$$\chi(\nu) \simeq \chi_0 - 2\Psi_0 \frac{\nu - \nu_0}{\nu_0}, \tag{2}$$

$$T(\nu) \simeq T_0 - \alpha T_0 \frac{\nu - \nu_0}{\nu_0}, \tag{3}$$

where χ_0 and T_0 are the values of $\chi(\nu)$ and $T(\nu)$ in the frequency ν_0. Let us introduce the denotation

$$-2\Psi_0 \frac{\nu - \nu_0}{\nu_0} = \xi \tag{4}$$

and assume for simplicity $\chi_0 = 0$. The Stokes integral parameters may be written in the form

$$I = \frac{\nu_0 T_0}{2\Psi_0} \int_{-\infty}^{\infty} F(\xi)\left(1 + \frac{\alpha\xi}{2\Psi_0}\right) d\xi = \frac{T_0\nu_0}{\Psi_0} \int_0^{\infty} F(\xi)\, d\xi; \tag{5}$$

$$Q = \frac{\nu_0 T_0}{2\Psi_0} \int_{-\infty}^{\infty} F(\xi)\left(1 + \frac{\alpha\xi}{2\Psi_0}\right) \cos(2\xi)\, d\xi = \frac{T_0\nu_0}{\Psi_0} \int_0^{\infty} F(\xi) \cos(2\xi)\, d\xi; \tag{6}$$

$$U = \frac{\nu_0 T_0}{2\Psi_0} \int_{-\infty}^{\infty} F(\xi)\left(1 + \frac{\alpha\xi}{2\Psi_0}\right) \sin(2\xi)\, d\xi = \frac{T_0\nu_0\alpha}{2\Psi_0^2} \int_0^{\infty} F(\xi)\, \xi \sin(2\xi)\, d\xi. \tag{7}$$

When passing to the final expressions (5) — (7), it was assumed that

$F(\xi)$ is an even function, decreasing sufficiently rapidly with the increase of ξ (this allowed to estimate the integration limits as infinite).

The position angle χ and the degree of polarization of radio emission P are linked with the Stokes parameters (5) ÷ (7) by the correlations *

$$\operatorname{tg}(2\chi) = U/Q, \qquad (8)$$

$$P = \sqrt{Q^2 + U^2}/I. \qquad (9)$$

In the case considered formulas (8) and (9) may be considerably simplified, since the parameter $U \ll Q$ so long as the degree of polarization of radio emission has a noticeable value. Let us illustrate this on examples of Π- type and Gaussian energetic frequency characteristics of the apparatus. In the example with the Π- type frequency characteristic $F(\xi) = 1$ at $-\Delta\Psi/2 \leqslant \xi \leqslant \Delta\Psi/2$ and $F(\xi) = 0$ at $|\xi| > \Delta\Psi/2$, where $\Delta\Psi$ is the difference in wave polarization planes' rotation angles, when the waves have the extreme frequencies in the apparatus' pass band $(\nu_0 \pm \Delta\nu/2)$. According to formulas (5) ÷ (7),

$$I = \frac{T_0 \nu_0 \Delta\Psi}{2\Psi_0},$$

$$Q = \frac{T_0 \nu_0}{2\Psi_0} \sin(\Delta\Psi),$$

$$U = \frac{T_0 \nu_0 a}{8\Psi_0^2} \sin(\Delta\Psi)\,[1 - \Delta\Psi \operatorname{ctg}(\Delta\Psi)].$$

The ratio

$$\frac{U}{Q} = \frac{a}{4\Psi_0}\,[1 - \Delta\Psi \operatorname{ctg}(\Delta\Psi)], \qquad (10)$$

at $\Psi_0 \gg 1$, $\Delta\Psi \sim 1$ and $\alpha \sim 3$ is much less than the unity, as may be easily seen, although the radiation depolarization is then already manifest on account of Faraday dispersion.

In the example with the Gaussian frequency characteristic of the apparatus $F(\xi) = \exp(-a^2\xi^2)$, where $a^2 = \nu_0^2 \ln 2/4\Psi_0^2(\Delta\nu)^2$ is the half-width of the pass band at the level 0.5 by power. In this case the ratio is

$$\frac{U}{Q} = \frac{a}{\ln 2}\,(\Delta\Psi)_{0,5}\,\frac{\Delta\nu}{\nu_0}, \qquad (11)$$

where $(\Delta\Psi)_{0,5} = 2\Psi_0\,\Delta\nu/\nu_0$, is also small at $(\Delta\Psi)_{0,5} \sim 1$, inasmuch as $\Delta\nu/\nu_0 \ll 1$.

* see for example [6].

Therefore, for the calculation of the polarization characteristics of the radiation we may use instead of formulas (8) and (9) the simpler correlations :

$$\chi = \frac{1}{2}\,\frac{U}{Q}; \qquad (12)$$

$$P = \left|\frac{Q}{I}\right| = \left|\int_0^\infty F(\xi)\cos(2\xi)\,d\xi\right|\left(\int_0^\infty F(\xi)\,d\xi\right)^{-1}. \qquad (13)$$

The expression (13) was utilized in the work [2] without detailed substantiation.

We compiled in Table 1 the results of calculations of the depolarizing factor, using the formula (13) for a series of frequency characteristics of the apparatus indicated in the first column of the Table. In the second column we brought out the expression for the normalized energetic characteristics of the apparatus. The quantity $d = (\Delta\nu)_{\text{эн}}/\nu_0$, where $(\Delta\nu)_{\text{эн}}$ is the energetic pass band width, is equal by the definition to $\int_0^\infty F(\nu)d\nu$.

In the third column of Table 1 we brought out the formulas determining the depolarization factor P. The quantity $\Delta\Psi$ is equal to difference in the rotation angles of waves' polarization planes, whose frequencies differ by $(\Delta\nu)_{\text{эн}}$, that is,

$$\Delta\Psi = 2\Psi_0\frac{(\Delta\nu)_{\text{эн}}}{\nu_0} = 4{,}8\cdot 10^4\, N H_\parallel\, l\,\frac{(\Delta\nu)_{\text{эн}}}{\nu_0^3}.$$

Here N is the electron concentration in the medium inducing the rotation; it must be expressed in cm^{-3}; and $H_{||}$ is the longitudinal component of the magnetic field expressed in oersted, l is the extension of the region occupied by the magnetoactive medium in the direction of radiation propagation, expressed in centimeters and ν_0 is the resonance frequency of the apparatus, expressed in cps. We have given in the last column of the Table 1 the ratios of the apparatus' pass band width at 0.5 level by power to the band's energetic width, allowing to pass in formulas for P from $(\Delta\nu)_{\text{эн}}$ to $(\Delta\nu)_{0,5}$, for usually $(\Delta\nu)_{0,5}$ is precisely the quantity being measured directly.

Figures 1 and 2 show how the degree of radiation polarization varies as a function of $\Delta\Psi$. If the apparatus' pass band is determined by several cascades tuned to a single frequency, for the computation of the depolarized

TABLE 1

Type of amplifier	Normalized frequency char.	Factor determining the depolarization, P	$(\Delta\nu)_{0,5}/(\Delta\nu)_{\text{эн}}$
One-circuit cascade	$\left\{1+\left[\frac{\pi(\nu-\nu_0)}{\nu_0 d}\right]^2\right\}^{-1}$	$\exp\left(-\frac{2}{\pi}\Delta\Psi\right)$	$\frac{2}{\pi}=0{,}65$
Two one-circuit cascades tuned to a single frequency	$\left\{1+\left[\frac{\pi(\nu-\nu_0)}{2\nu_0 d}\right]^2\right\}^{-2}$	$\exp\left(-\frac{4}{\pi}\Delta\Psi\right)\left(1+\frac{4}{\pi}\Delta\Psi\right)$	$\frac{4}{\pi}(\sqrt{2}-1)^{1/2}=0{,}81$
Four one-circuit cascades tuned to a single frquency	$\left\{1+\left[\frac{5\pi}{16}\frac{(\nu-\nu_0)}{\nu_0 d}\right]^2\right\}^{-4}$	$\exp\left(-\frac{32}{5\pi}\Delta\Psi\right)\left[1+\frac{32}{5\pi}\Delta\Psi+\frac{2}{5}\left(\frac{32}{5\pi}\right)^2(\Delta\Psi)^2+\frac{1}{15}\left(\frac{32}{5\pi}\right)^3(\Delta\Psi)^3\right]$	$2(\sqrt[4]{2}-1)^{1/2}=0{,}89$
Gaussian frequ characteristic	$\exp\left\{-\left[\frac{\sqrt{\pi}(\nu-\nu_0)}{\nu_0 d}\right]^2\right\}$	$\exp\left[-\frac{(\Delta\Psi)^2}{\pi}\right]$	$\frac{2\ln 2}{\sqrt{\pi}}=0{,}94$
Two mutually detuned cascades	$\left\{1+\left[\frac{\pi}{\sqrt{2}}\frac{(\nu-\nu_0)}{\nu_0 d}\right]^4\right\}^{-1}$	$\left\lvert\exp\left(-\frac{2}{\pi}\Delta\Psi\right)\left[\cos\left(\frac{2}{\pi}\Delta\Psi\right)+\sin\left(\frac{2}{\pi}\Delta\Psi\right)\right]\right\rvert$	$\frac{2\sqrt{2}}{\pi}=0{,}9$
Three mutually detuned cascades	$\left\{1+\left[\frac{2\pi}{3}\frac{(\nu-\nu_0)}{\nu_0 d}\right]^6\right\}^{-1}$	$\left\lvert\frac{1}{2}\left\{\exp\left(-\frac{3}{\pi}\Delta\Psi\right)+\exp\left(-\frac{3}{2\pi}\Delta\Psi\right)\left[\sqrt{3}\sin\left(\frac{3\sqrt{3}}{2\pi}\Delta\Psi\right)+\cos\left(\frac{3\sqrt{3}}{2\pi}\Delta\Psi\right)\right]\right\}\right\rvert$	$\frac{3}{\pi}=0{,}96$
Two pairs of mutually detuned cascades	$\left\{1+\left[\frac{3\pi}{4\sqrt{2}}\frac{(\nu-\nu_0)}{\nu_0 d}\right]^4\right\}^{-2}$	$\left\lvert\exp\left(-\frac{8}{3\pi}\Delta\Psi\right)\left[\sin\left(\frac{8}{3\pi}\Delta\Psi\right)+\cos\left(\frac{8}{3\pi}\Delta\Psi\right)\right]+\frac{16}{9\pi}\Delta\Psi\exp\left(-\frac{8}{3\pi}\Delta\Psi\right)\sin\left(\frac{8}{3\pi}\Delta\Psi\right)\right\rvert$	$\frac{8\sqrt{2}}{3\pi}(\sqrt{2}-1)^{1/4}=0{,}96$
Two triplets of mutually detuned cascades	$\left\{1+\left[\frac{5\pi}{9}\frac{(\nu-\nu_0)}{\nu_0 d}\right]^6\right\}^{-2}$	$\Big\lvert\frac{1}{2}\left(1+\frac{18}{25\pi}\Delta\Psi\right)\exp\left(-\frac{18}{5\pi}\Delta\Psi\right)+\frac{1}{2}\left(1+\frac{18}{50\pi}\Delta\Psi\right)\exp\left(-\frac{1}{2}\frac{18}{5\pi}\Delta\Psi\right)\left[\sqrt{3}\sin\left(\frac{9\sqrt{3}}{5\pi}\Delta\Psi\right)+\cos\left(\frac{9\sqrt{3}}{5\pi}\Delta\Psi\right)\right]-\frac{0{,}9\sqrt{3}}{5\pi}\Delta\Psi\exp\left(-\frac{9}{5\pi}\Delta\Psi\right)\left[\sqrt{3}\cos\left(\frac{9\sqrt{3}}{5\pi}\Delta\Psi\right)-\sin\left(\frac{9\sqrt{3}}{5\pi}\Delta\Psi\right)\right]\Big\rvert$	$\frac{18}{5\pi}(\sqrt{2}-1)^{1/6}=0{,}99$
π-type frequency characteristic	1 at $\lvert\nu-\nu_0\rvert<\frac{(\Delta\nu)_{\text{эн}}}{2}$ 0 at $\lvert\nu-\nu_0\rvert>\frac{(\Delta\nu)_{\text{эн}}}{2}$	$\left\lvert\frac{\sin\Delta\Psi}{\Delta\Psi}\right\rvert$	1

radiation on account of the Faraday effect dispersion the frequency characteristic of the apparatus may be approximated by the Gaussian curve. If there are several groups of mutually-detuned cascades, the П -type characteristic is quite satisfactory. Let us underline that at strong radiation depolarization ($P \leqslant 0.05$) the values, computed according to formulas of Table 1 and plotted in Figs. 1 and 2, may strongly differ from the exact values, which ought to be computed by the formula (9). However, these exact values of P do not offer interest because of the smallness of the degree of radiation polarization.

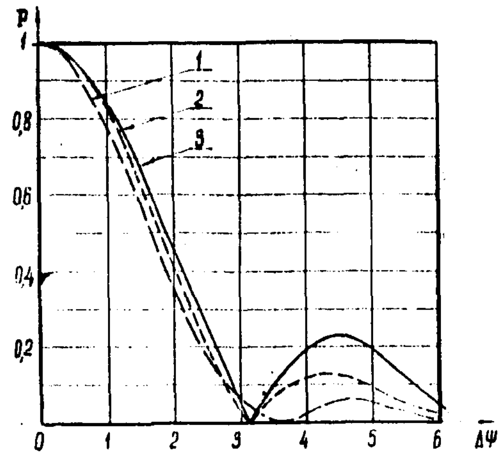

Fig. 1. - Dependence of the degree of polarization P on $\Delta\Psi$:

1 - two pairs od disturbed cascades; 2 — two triplets of same; 3 — П - type frequency characteristic.

Let us still pause on the question of the position angle's dependence on the apparatus's pass band width. As may be seen from formulas (10), (11) and (12), the variations of the angle χ at values $P \geqslant 0.05$ and $\Delta\nu/\nu_0 \sim 10^{-1}$, do not exceed a a few degrees. Thus, the dependence of χ on $\Delta\nu$ may usually be neglected.

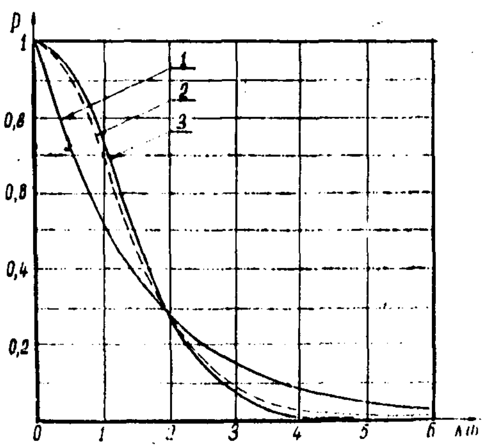

Fig. 2. - Dependence of the degree of polarization P on $\Delta\Psi$:

1 — one-circuit cascade; 2 — four one-circuit cascades, tuned to a single frequency; 3 — Gaussian frequency characteristic.

***** THE END *****

Radiphysical Institute of Scientific Research at Gor'kiy University.

Received on 25 June 1965

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Contract No. NAS-5-9299
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Translated by ANDRE L. BRICHANT
on 21 - 22 February 1966